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PORTFOLIO REBALANCING –
Evidence on Finnish Household Investors

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Abstract

This study concentrates on the phenomenon of portfolio rebalancing. Portfolio rebalancing is defined as a process, in which an investor liquidates some part of her portfolio, and uses the proceeds to acquire another asset or assets. After studying existing literature, I use the data of the central register of share holdings for Finnish stocks for quantitative analysis. The main findings are that the trades of the household investors are strongly clustered, and that around 24.7% of the transactions take place in a cluster including both buys and sells. Furthermore, these clusters including both buys and sells seem to be speculative. When an investor sells some of her holdings in one stock, she very often sells it entirely. Thus, on average, the rebalancing activity does not have effect on the mean-variance ratio of the portfolio measured by the Sharpe Ratio.

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I. Introduction

The portfolio rebalancing is an interesting phenomenon for several reasons. Firstly, it is largely ignored part of the financial theory. Taking it into account could improve scientific analysis on, and modelling of the financial markets. Secondly, empirical description of investors' behaviour on the subject would help financial institutions to offer their clientele appropriately priced and structured services to manage their portfolios. Thirdly, good models on rebalancing and tools based on these models could help investors to make better-judged decisions on how to manage their investments. Finally, as all the things helping us to understand surrounding world and describing our own behaviour, it is interesting per se.

On the scientific field the process of rebalancing and the effect of rebalancing on the trading volume are not thoroughly analysed. Odean (1999) states that there are not good economic models to predict what the trading volume on the markets should be. Portfolio rebalancing certainly cannot be ignored in these models. There are theoretical models describing rebalancing, (e.g. Constantinides 1986, Morton and Pliska 1995), but their implications on market behaviour and trading volume are unclear.

In the empirical studies Karhunen & Keloharju (2001) provide good overview on shareownership in Finland, including information on portfolio diversification. Recent studies by Grinblatt and Keloharju (2000, 2001a) have comprehensively analysed determinants of buy and sell transactions. These studies cast light on investors' stock holdings and trading behaviour, but concentrate on individual transactions. Therefore they omit the effects on the composition of the portfolio.

On the practical side fund managers, investors and their advisers need to consider from time to time does it pay to change the composition of the portfolio. The financial theory provides methods to determine the feasible composition of the portfolio, but has lot less to offer in advising how far from this target

portfolio should an investor let her portfolio drift, before taking costly measures to change the weights by trading.

There is not one exact definition for term “portfolio rebalancing”. According to Wall Street Journal Europe (11.2.2002) “The idea is that investors should have an overall plan for the portions of their portfolios to be devoted to various types of mutual funds or other securities. Then, as market prices change, they should periodically buy or sell some of those securities to bring the portfolio back in line with the target mix.”

In this study, I use somewhat broader definition of rebalancing, which requires neither periodical trading nor overall plan. Instead, it includes all trading, which is done in purpose to alter the proportions of assets in investor’s portfolio. More formally I define:

Portfolio rebalancing is a process, in which an investor liquidates some part of her portfolio, and uses the proceeds to acquire another asset or assets.

Prior research and this study

Prior research on the portfolio rebalancing has been theoretical. The models presented in the literature (see later chapters) differ considerably in their assumptions, and do not give clear picture on what we should expect to observe on the stock market.

To my knowledge, there is no prior empirical research on portfolio rebalancing. This is somewhat puzzling, since it has often been mentioned as one of the potential reasons for trading (Lakonishok and Smidt 1986, Odean 1999, Grinblatt and Keloharju 2001a). One obvious reason for this is the lack of suitable data. Odean (1999) claims that “there are three data sets similar to this one [i.e. his data set] in literature”. The set I use does not belong to Odean’s list, and it is therefore the fifth one. It has been used for several studies in recent years (e.g. Grinblatt and Keloharju 2000, 2001a, Karhunen and Keloharju 2001, Ekholm 2002)

This paper aims to give some indication on how do investors rebalance in practice. This would help the researchers in formulating models that are better in describing what we observe in practise, and which may provide advice for practitioners. Further empirical research is however needed to capture the nature of this phenomenon.

The research questions

The research questions I try to answer in this study are:

- 1) *From a theoretical point of view, why and how should an investor rebalance his portfolio?*
- 2) *What is the practise of rebalancing in the Finnish stock market?*

First question is addressed in the review of theory, and the second one in the empirical part. The emphasis is on the latter question.

The limitations of the study

There are some limitations in this study. Firstly, I study only on the wealthy household investors, and many results are likely to be specific to this group. Reasons for this choice are discussed at the beginning of the empirical part, most important being the stronger results that can be achieved by using more homogenous dataset.

Secondly, other assets than common stocks are excluded. This is due to the lack of data on other investments. This limits the study to concentrate on the changes the investor makes within that proportion of the portfolio, which is invested in stocks. However, though this limits the area that can be studied, it is not likely to bias the results.

Thirdly, the data used is on Finnish investors, and it is possible that some aspects are characteristic to this group. Only further research can verify which

are the differences to other countries. I however assume that most of the conclusions made can be generalised.

II. Review of Theory

Everyone knows the saying “don’t put all your eggs in one basket”. However, when it comes to investments, much more can be said on how an investor should divide her eggs between different baskets, and when, if ever, should she pick an egg from one basket and place it in another.

At the first section of this chapter I present theories on portfolio selection. These theories aim to answer how an investor should initially divide her wealth between different investment opportunities, and what should her investment objectives be.

Next I move on to models on portfolio rebalancing. The models are classified according to their underlying transaction cost structure. The final section summarises some of the central results of this chapter and discusses their relevance for the empirical part of this study.

Portfolio Selection

The Markowitz Model

Harry Markowitz (according to Luenberger (1998) in *Journal of Finance* 7, No 1, 77-91, 1957) defined what is known as a “mean-variance optimising investor”.

This means that an investor chooses some expected mean return on her investment, and then minimises the expected variance of this return.

Mathematically he formulated the problem as follows:

$$\text{Minimise} \quad \frac{1}{2} \sum w_i w_j \sigma_{ij}$$

$$\text{Subject to} \quad \sum w_i r_i = r$$

$$\sum w_i = 1$$

The problem can be solved analytically if short selling is allowed. If it is not, the solution can be found by quadratic programming (Luenberger 1998, pages 158-161).

The great insight this problem provided is that an investment cannot be evaluated based only on its expected return, but also the riskiness (i.e. variance) of that return must be considered. In addition, the inclusion of the covariances indicates that it is not enough to know how individual assets behave, but how they behave together. All in all, as Luenberger writes, "it [the model] makes the trade-offs between mean and variance explicit".

Tobin (according to Luenberger in *Review of Economic Studies* 26, Feb, 65-86, 1958) developed Markowitz's model further by including a risk-free asset. This led to "One-Fund Theorem". The theorem states "There is a single fund F of risky assets such that any efficient portfolio can be constructed as a combination of the fund F and the risk-free asset."

The assets included in fund F and their weights remained a question. One might think that this problem could be solved by crude computing power. However, Luenberger (1998, page 218) shows that "it is fundamentally impossible to obtain accurate estimates of expected returns of common stocks using historical data. The standard deviation (or volatility) is just too great. Furthermore, the solution of the Markowitz mean-variance portfolio problem tends to be fairly sensitive to these values."

The aim to optimise utility of return-variance relation is nowadays commonly accepted objective in evaluating the investment decisions. This holds outside academic circles as well, which can easily be verified by browsing through homepages of investment banks and mutual funds, which present wide variety of graphs and tables describing their investment policies.

The Capital Asset Pricing Model

The capital asset pricing model (CAPM) is logical continuation to Markowitz model and it was developed independently by Sharpe (1964), Lintner (1965, Mossin (1966) and Treynor (1961). (Luenberger 1998)

The model requires that the investors have homogenous beliefs and that the markets are frictionless. The solution of one-fund theorem is derived from this. Under these assumptions everyone holds the same portfolio of risky assets, and this portfolio must equal the market portfolio, i.e. the portfolio where the weight on each asset is the market value of that asset divided by the market value of all risky assets. (Grinblatt and Titman 1998, pages 166-167).

The implication of this theory on portfolio selection is relatively straightforward: it does not pay to choose stocks; the optimal investment strategy is just to buy the market portfolio and hold it until the end of the investment period.

The Arbitrage Pricing Theory

Clearly, the assumptions of the perfect markets are not met in the real-life. Investors have differing beliefs, and there are considerably frictions on the markets. Various studies have extended CAPM by accommodating different frictions (i.e. absence of riskless asset, no-short sales, see Grinblatt & Titman 1998, pages 171-172 for references).

Several factor models were also developed to improve the accuracy of CAPM (which basically is one-factor model, where market-portfolio is the only factor). Most important of these is the arbitrage pricing theory (APT) developed by Ross (1976). The APT has less restrictive assumptions than CAPM does. These assumptions are 1) returns can be described by a factor model 2) there are no arbitrage opportunities and 3) there is a large number of securities. (Grinblatt and Titman 1998, page 218)

Assumptions of the APT seem to be more realistic than those of the CAPM, and they for example allow the investors to hold differing portfolios. Consequently, whereas CAPM has done relatively poorly in empirical tests, APT has not been completely rejected. (Grinblatt and Tittman pages 177-186 and 226-228).

It should be noted, that the studies cannot directly test the models, but rather their implications on markets. In addition they include parameters that must be estimated instead of observing (most notably the composition of the market portfolio in CAPM). Thus, the validity or invalidity of these models remains controversial.

Both CAPM and APT have a common implication on securities prices: if they hold at least approximately, the share prices reflect companies' fair values, and the return of the portfolio cannot be improved by actively managing portfolio (i.e. picking the shares). Well known hypothesis of information-efficient markets has the same implication.

The reality seems to support the existence of fair values if the information search costs are included (although several studies have reported so called "market anomalies"). Carhart (1997) states that "Persistence in the mutual fund performance does not reflect superior stock picking skill. Rather, common factors in stock returns and persistent differences in the mutual fund expenses and transaction costs explain almost all of the predictability in mutual fund returns. Only the strong, persistent underperformance by the worst-return mutual funds remains anomalous."

Carhart's conclusion means that on average, despite highly trained professionals and billions of dollars invested on sophisticated analysis on stocks, the investment funds can at best match the performance of each other. It is however possibly to fare worse than this.

Inclusion of Transaction Costs

One important friction on the real world markets are the transaction costs. These include service charges, commissions, bid/ask spreads, the time required for the transaction (Hess 1991), market impact costs, transaction taxes (Vayanos 1998), cost of analysis, information search cost and any expense incurred in the process of deciding upon and placing an order (Dumas and Luciano 1991).

Brennan (1975) points out that the normative theory of portfolio selection is in most part based on assumption that there are no costs of transacting in securities markets. In his illustratively named paper *"The optimal number of securities in a risky asset portfolio when there are fixed costs of transacting: theory and some empirical results"* he shows that the introduction of fixed cost of transacting will have the effect of "reducing substantially the optimal number of securities to include in an investor's risky asset portfolio, at least at moderate wealth levels".

The paper handles single-period mean-variance investment problem, where investor is assumed to initially divide her wealth between risky stocks and riskless asset, and then to hold this portfolio. He assumes in his framework that the securities are "properly priced in that their ex-ante expected returns conform to the predictions of CAPM", i.e. the rewards of the share picking are negligible.

Based on this he develops two models. First one is simple but illustrative model where the returns of all securities have same systematic risk and residual variance. Second model is more general, and it allows for the varying systematic risk. Finally he uses data of all continuously listed 593 securities from CRSP tape for the period 1946-65 to illustrate the implementation of the latter model.

For the fixed cost of transaction of \$10 per security, and the wealth level of \$5000, the simpler model implies 5 as the optimal number of securities in the portfolio, and the more complicated model only 2. For the wealth of \$10000 the

corresponding values are 8 and 3. (There are slight differences in other parameters of these solutions as well, for example amount invested in stocks varies from 100% to 116%)

Brennan claims that statistical reasons cause the more complicated model to underestimate the optimal number of securities, and the assumptions of the simpler model to overestimate the optimal number. Thus, the real number (based on this model) is somewhere between these. Note that the \$10 transaction fee generates (same order of solutions as above) 1,0%, 0,4%, 0,8% or 0,3% expenses in relation to the initial wealth, which seems to be of realistic magnitude for private investor.

Portfolio Rebalancing

When investor has purchased an initial portfolio, the prices of securities forming it start to evolve. This causes changes in the relative proportions of the securities. If investor wants to maintain or restore the original proportions, she needs to trade.

Let us consider as an example a portfolio consisting of 1/3 in stocks A, B and C. After one year, stock A has appreciated 55%, B depreciated 32% and C appreciated 12%. The total value of the portfolio has grown modest 11.7% during the period. The weights of the stocks, which form the portfolio, have changed more dramatically and are now 46.3%, 20.3% and 33.4%. Clearly, the portfolio is now much more sensitive to the fluctuations of stock A.

The problem is that it is difficult to say should the investor rebalance the portfolio now, should she wait, or had it been wise to do the rebalancing already earlier. This clearly depends at least on the cost of rebalancing, on the utility the investor places on the portfolio consisting on the 1/3 proportions instead of 46.3-20.3-33.4 proportions and on the stock and market characteristics. The models presented in this section try to provide answers on such questions.

The theories developed to model portfolio rebalancing can be divided in two categories according to the underlying transaction cost structure. The first line assumes that the costs are proportional to the value of the trade (e.g. Constantinides 1986, Dumas and Luciano 1991). Second, less studied, line concentrates on models with fixed transaction cost component (e.g. Morton and Pliska 1995).

Trading on the real market naturally causes both types of costs. However, since the models are relatively complicated and the solutions require extensive computation, this division must be accepted.

I assume that the models with proportional costs are more realistic in describing financial institutions. When they trade, the fixed cost is small in relation to trade sizes, and the largest costs are perhaps related to bid-ask spread and market impact, which are proportional in nature.

The models with fixed cost component are better suited to describe private investors' cost structure. Their trades do not cause noticeable market impact, and the effect of the bid-ask spread is smaller than the service fees and commissions, which usually contain both proportional and fixed component. Maybe the largest cost component is however the time required for information search and completing the trade (which is closer to fixed than proportional).

Barber and Odean (2002) report the magnitude of transaction costs for the clients of a U.S. discount brokerage house in period 1991-1995. In the phone-based trades of more than \$1000, the average commission on the value of the trade was 1.64% for the buys and 1.47% for the sells. The corresponding bid-ask spread components of transaction costs were 0.32% and 0.76%. For a round trip, these costs total 4.19%.

The new technology has recently pushed the fixed components downwards, but they are still very considerable for private investors.

Rebalancing - Models Without Transaction Costs

One of the first and most cited papers on portfolio rebalancing was written by Merton (1971). In his model the prices of the shares follow geometric Brownian motions and the investors trade without costs. The investment portfolio consists of a single riskless bond and n risky stocks.

The investment strategy is described by the fraction of investor's total wealth she holds in each of the assets. For example, in a case of two assets, the optimal strategy can be given as fraction π of the total wealth invested in a risky stock and fraction $(1-\pi)$ in a riskless bond.

Merton and Pliska (1995) point out that since the proportions start to change when the prices evolve even if the investor does not trade, she needs to trade constantly in order to keep the portfolio weights constant. Under Merton's assumptions this [keeping the weights constant] indeed is the optimal strategy.

Rebalancing - Models With Proportional Transaction Costs

Proportionality [of transaction costs] means that the cost incurred is proportional to the value of the trade (Dumas and Luciano 1991).

Constantinides (1986) studies an investor, who maximises the expected utility of infinite horizon consumption stream. He assumes that the investor divides her wealth between risky and riskless asset, and consumes some fixed proportion of his wealth in each period. In his model the trades are elegantly determined endogenously. He states that this is an important improvement compared to some earlier models on this category, in which investors arrival to market is determined exogenously.

Constantinides solves his model numerically, and the solutions imply that the investors drastically reduce the frequency and volume of the trading when the transaction costs are introduced. Investment policy is characterised by a region

of no transactions. The investor does not trade as long as the ratio of asset values lies in this wide interval.

He compares the results of his model with transaction costs to those without them, and finds that in addition to reduction in trading, these costs produce a liquidity premium in magnitude of 0,1 to $0,25 \cdot (\text{transaction cost \%})$ p.a., and therefore have only second order effect on equilibrium asset returns. Thus, the CAPM can at least theoretically be used as a pricing model notwithstanding transaction costs.

Constantinides claims that the extension of his model to allow for more than one risky asset is in principle straightforward. "The computational requirements, however, are enormous." He guesses that the introduction of new securities with same variance would lead to the drop of the liquidity premium.

Whereas Constantinides (1986) solved his model numerically, Dumas and Luciano (1991) derive an exact solution to their essentially similar problem. In their model investor does not consume along the way, but instead accumulates wealth until some terminal point in time when she consumes all. They postpone this point infinitely far into the future to obtain stationary portfolio rule.

In their paper Dumas and Luciano test this model by changing various parameters (risk-aversion, transaction cost level, increasing risk) and get qualitatively the same results as those of Constantinides. However, the no-transaction area is even wider, and the increase in transaction cost neither bias the portfolio towards riskless asset [as Constantinides claimed in his paper], nor away from it.

Morton and Pliska (1995) criticise the models where the costs are linear in value of stock traded. They claim that "the typical optimal policy under this transaction cost regime (in the case of the single risky stock) is to make no trades when the fraction of portfolio held in the risky stock lies in a certain 'no-trade' interval. When the fraction reaches either end point of this interval, the investor makes infinitesimal trades to keep the fraction on the border of, or inside, the

interval. In other words, under this transaction cost model, we again find investors trading continuously at least part of the time [as they do in Merton's no transaction cost model]." Unsurprisingly, their own model operates in fixed transaction cost environment.

Rebalancing - Models With Fixed Transaction Costs

Morton and Pliska (1995) study a transaction cost structure, where the cost of every trade is equal to a fixed fraction of the value of the entire portfolio. According to them, this has been termed as 'portfolio management fee' by Duffie and Sun (*J of Econ. Dyn. Control* 14, 35-51, 1990).

The investor in Morton-Pliska model does not consume along the way, but instead maximises asymptotic rate of growth of the portfolio value. They state that "the induced portfolio behaviour under the growth rate criterion should be similar to that of an investor with logarithmic utility of wealth at a distant terminal horizon".

The investor allocates her wealth in the proportions b_0 at some time t_0 and then, as the prices evolve, stops at some point of time t_1 to rebalance her portfolio. At that point, the portfolio management fee is subtracted from the portfolio value, and the proportions change to b_1 . After that, the prices continue to evolve. The investment problem is to choose b and t so that the growth rate is maximised.

They solve the model analytically in the case of one risky stock, and numerically in the two stock case. In both cases, the optimal policy is always (if other parameters have not changed) to return the weights to b_0 . The solutions of one and two stock models do not differ qualitatively from each other.

The optimal policy is not to transact as long as the weights are in no-transaction area, (or continuation region, as Morton and Pliska term it). The size of the transaction cost is the major component in defining expected time before the weights drift outside it, and the weights are returned to b_0 . Other factors seem to have only minor effect. The expected time is fairly long, in the region of two to

three years with a fee of 1%, and around 7 years with 5% fee. The region in which the asset weights may fluctuate is correspondingly wide.

The model also suggests that the optimal weights are almost the same as those of no-transaction cost case, i.e. the Merton-model. Since their model is burdensome to solve, Morton and Pliska suggest that the investor should concentrate on Merton's solution. In addition, the investor should avoid, as much as possible, paying a costs to transfer funds among stocks. They assume that in practical situations inflow to, and outflow from portfolio might be sufficient to keep portfolio proportions near b_0 .

These recommendations are however dependent on the presumed transaction cost structure. The assumption that the fee would be totally fixed seems somewhat unrealistic. Inclusion of some proportionality would probably reduce the time between rebalancings. The comparison of this model to those with totally proportional transaction cost structure can offer invaluable insight.

Atkinson and Wilmott (1995) have developed an extension to Morton-Pliska model, which trades off little accuracy to explicit solutions for the optimal trading policy. They claim that this makes the solutions for realistically large number of assets a practical possibility.

Conclusions on the Theory

By now it seems clear that the available theoretical studies can give only very vague answers on the practice of the portfolio rebalancing. However, they give some important guidelines, and facilitate the concentration to relevant themes in the empirical part of this study.

Does theory support the existence of rebalancings?

Firstly, Brennan's study (1975) indicates that for a private, mean-variance optimising investor it is optimal to hold poorly diversified portfolio [, if the transaction costs are present, and the portfolio is small]. This result holds with

common sense. In addition, many Finnish individual investors seem to hold such portfolios even in the late 1990's when wide variety of mutual funds was available [Although I do not discuss the rationale for this here].

According to Karhunen and Keloharju (2001) 56,2% of Finnish private investors held in June 2000 only one stock in their portfolio, and the average number of stocks held was 2.4. The larger portfolios (according to their value) consisted of more stocks. According to the same study household investors with at least one million FIM worth of shares held on average 9.3 stocks, although even of them 5.5% held only one stock. It is reasonable to think that on average, the portfolios consisting of more stocks are better diversified.

Secondly, all the models regardless of the transaction cost structure and other varying assumptions indicate that an investor [holding poorly diversified portfolio] needs to rebalance her portfolio from time to time. The interval varies depending on model from infinitesimal small (Merton) up to seven years (Morton and Pliska). The need for rebalancing arises from portfolio's deviation from optimal mean-variance ratio.

These facts support the belief, that there should occur rebalancings on the market, though Morton and Pliska suggest that an investor should try to avoid them if possible by using in- and outflows of funds for rebalancing.

How do pre-rebalancing portfolio and different variables affect the rebalancing?

If underlying factors have not changed, the optimal investment rule remains constant during investor's investment horizon in all of the models. That is, the investor should follow some rule like "rebalance this way after this ratio has exceeded 0.37." This optimal rule differs from model to model. It is however unlikely that the conditions on the markets are stable enough.

Campbell et al. (2001) have studied the changes in the volatility and in the correlations among stocks. They find that the market level volatility remained

relatively stable in period 1962-1997 whereas firm level volatility has increased strongly, and the correlations among individual stocks have declined. They claim that “the declining correlations among individual stocks imply benefits of portfolio diversification have increased over time”, but the “increase in idiosyncratic risk has increased the number of stocks needed to reduce excess standard deviation to any given level.” This kind of change, though slow, clearly shows that the optimal solution given by the rebalancing model is subject to changes in market factors.

The models differ in their relation between pre- and post-rebalancing portfolios. In the models with proportional transaction cost structure the optimal portfolio after rebalancing is close to investor’s pre-rebalancing portfolio, whereas in Morton-Pliska model the portfolio is “reset” to its original composition. A realistic assumption might be that when both types of expenses are present, the portfolio would somewhat lean towards pre-rebalancing portfolio.

Morton and Pliska state in their conclusions that their model generalises nicely to time varying drift and diffusion coefficients, which causes the optimal “Merton proportions” to change with time. In real-life applications these varying proportions would be used instead of original proportions.

Thus, the theory does not give clear answer on how ex-ante proportions and the changes in environment affect the optimal portfolio and rebalancing rules.

III. Empirical Part

In the empirical part I study trades completed by Finnish wealthy private investors in years 1995-2000. The data used allows me to observe each transaction of each investor during the period. This makes it possible to study series of transactions as an entity, in contrast to studying them separately as often is done.

The methodology is based on the idea of clustering the individual trades of an investor together, and to use these clusters instead of the individual transactions in the analysis.

I have combined investor's transactions taking place close to each other together to form a cluster. I have then distinguished between those clusters, which includes both buy and sell transactions (e.g. investor sells shares worth FIM50.000 on Monday, and buys shares worth FIM45.000 on Thursday), and to those clusters, which do have trades only to one direction.

Chan and Lakonishok (1995) have used similar methodology of "forming packages" of money managers' buy or sell transactions on a single stock instead of studying them on individual level.

Disposition of the Empirical Part

At the beginning of the empirical part I define the terminology and present the dataset used for this study. After that I move on to form the clusters, on which the rest of the study is based.

The first point of interest is the magnitude of the rebalancing activity and the properties of the clusters.

Next I move on to study how the rebalancing is done. This includes studying how large proportion of the portfolio is changed during the rebalancing and which factors affect this proportion. Furthermore, the effect of the rebalancing on the number of different stocks held in the portfolio is studied.

Following point of interest is how often an investor rebalances her portfolio, how this is related to the number of another clusters completed by this investor and whether the intervals between the rebalancings are frequent.

The chapter is completed by the section where I inspect how the mean and the variance of the portfolio change during the rebalancing.

Terminology

In the empirical part, the following terminology is used:

Transaction: Any register entry in the FCSD database, which actually changes the portfolio held by an investor (e.g. trades and IPOs). Stock splits are not included in this category.

Trade: A purchase or sale, which takes place on the exchange. These are marked with code 01 in FCSD data.

Cluster: One or more trades, which take place close to each other, and which are separated from other trades with a clear break. Different cluster categories are explained in section “Forming the Clusters”.

Rebalancing: Portfolio rebalancing is a process in which an investor liquidates some part of her portfolio, and uses the proceeds to acquire another asset or assets. Only intra-cluster trading is excluded (i.e. Those clusters that contain both buys and sells with one stock). Otherwise, the motive is not considered (i.e. clusters made both for longer term speculation and for improving mean-variance ratio are included).

Value of the cluster: (Amount) x (Price) of every transaction summed together, i.e. for rebalancing clusters the value is the sum of both buys and sells.

Description of the Data

In this study I use the data from the Finnish Central Securities Depository (FCSD). This register covers more than 97% of the total market capitalisation of Finnish stocks as of beginning of 1995 (Grinblatt and Keloharju 2000). It includes initial ownership in January 1995, and all subsequent changes until the end of May 2000. Based on this information, the composition of every individual portfolio at the end of each day during this period can be calculated.

I have included only those investors classified as household investors in this study (6 categories). Other 36 categories, which include for example financial institutions, general government, international organisations, European Union and non-financial corporations, are ignored.

I have chosen to study wealthy private investors for number of reasons.

- 1) Private investors do large proportion of all trading, and thus this group is economically significant.
- 2) Trading of these investors is not too frequent. For example for financial institutions forming of the clusters would have been much more troublesome.
- 3) Choosing only wealthy investors reduces sample, and thus computing power needed.
- 4) Lower limit is also set because I assume that the investors owning smaller portfolios are less likely to manage them carefully. These portfolios also consist of less stocks (Karhunen & Keloharju 2001)
- 5) Upper limit: The aim of the upper limit is to keep the portfolios observed so similar that the size related factors do not affect the results too strongly. There are no earlier studies to benchmark the results to, so the setting needs to be simple. The portfolios exceeding upper limit are likely to be more professionally managed, better-

diversified etc. See Grinblatt & Keloharju (2001a) for the differing behaviour of household investors according to their wealth level.

The raw data consists of portfolio information of 484,919 Finnish private investors, who owned Finnish shares 1.1.1995. First I have excluded non-wealthy investors, whose portfolio's value was less than FIM 210,000, and rich investors, whose portfolio's value was over FIM 900,000 (ca. 35,000€ and 150,000€ respectively ¹). This leaves me 16,909 wealthy investors. Next I removed 2,621 passive investors, who did not trade at all in the Helsinki Exchanges between 1.1.1995 and 31.5.2000. The changes in the portfolios of the remaining 14,288 investors are the sample for my study. Thus, the data used consists of

The portfolio changes of 14,288 Finnish private investors, who committed at least one trade in Helsinki Exchanges between 1.1.1995 and 31.5.2000, and whose portfolio value was between FIM 210,000 and 900,000 as of 1.1.1995.

The FCSD data is supplemented with the closing prices of the Helsinki Exchanges for the same period and with the corresponding split and dividend adjusted return data.

Primarily, I have used the prices from the FCSD data where possible (e.g. all the individual transactions, the value of buys or sells in the clusters). However, if the calculation has required some price information that is not available in the FCSD data, I have consistently used Helsinki Exchanges prices/returns for entire calculation (e.g. value of the portfolio before the start of the cluster usually includes stocks that are not traded in that cluster, and thus I have used the preceding Helsinki Exchanges closing prices for all the stocks included into portfolio). In most cases the choice is evident. Where not, I have indicated it.

¹ The values are counted using portfolios 1.1.1995, but the prices are closing prices for 2.1.1995, since the New Year's Day was not trading day. I assume that the difference is trivial, and the computing effort needed to count portfolios 2.1.1995 would have been disproportionate to improvement achieved in the sample.

Forming the Clusters

I start my analysis by forming the clusters of the transactions of the individual investors. These clusters are classified in five categories; pure buys, pure sells, rebalancings, sells including rebalancing and purchases including rebalancing.

The clustering is based on the simple idea of an investor not making her trades transaction by transaction, but instead having an overall plan, which she takes into account when making decisions. This idea is operationalised as follows;

The trades of an individual investor are clustered together if there are not long pauses between them. However, there must be a pause before and after such a cluster. More accurately, I have set the maximum pause between trades to be 14 natural days, and require at least 14 natural days break before and after the cluster formed by these trades. (*e.g. from the trades on days 1, 9, 12 and 29 first three are clustered together*)².

To keep the data coherent, I have next excluded long clusters, i.e. those that last over 20 trading days. The reason is that it is more difficult to determine do all of these trades belong together and what causes them ³. Later when studying durations of the clusters we see that only approx. 1% of the clusters finally included last 15 days or more, so the exact limit, be it 15 or 20 days, is likely to be unimportant.

Next the clusters including day trading or rather “intra-cluster trading” (i.e. buy and sell transaction for the same security in the same cluster) are excluded. These trades do not satisfy the condition that “the trading... is done in purpose to alter the proportions of assets in investor’s portfolio” (introduction of this study), since these changes are at least partially reversed almost instantly.

Finally, all the clusters including non-exchange transactions or other register entries (e.g. shares bought in IPO or SEO, stock split) are excluded. Firstly, these

² Actually, the date used in this phase is the date when the transaction was entered into the registry. After the clusters have been formed, these dates are replaced with trade dates.

would potentially cause errors in analysis, and since there is no reason to believe that their exclusion would qualitatively bias the results, I decided to exclude them altogether.

Secondly, participating in IPOs and SEOs is not as clear-cut process as trading on the exchange. The investor typically must make the decision of participation at least days, usually weeks before she actually receives stocks. Private investors are also often required to make the payment in advance. Furthermore, the number of stocks to be received and the total value of the investment are often unknown. Thus, it would have been much more difficult to incorporate IPOs and SEOs into analysis.

The remaining clusters will be used in the analysis. They are classified as follows:

Pure buys: A cluster contains only buys.

Pure sells: A cluster contains only sells.

Rebalancings: In a cluster, the total value of the buys is within the range of 75% to 125% of the sells. This is selected so that most of the proceeds are reinvested, and that the new investment is a minor component.

Sells including rebalancing: Cluster that contains both buys and sells, but the amount reinvested is less than 75%.

Purchases including rebalancing: Cluster that contains both buys and sells, but the total value of buys is more than 125% of the total value of the sells.

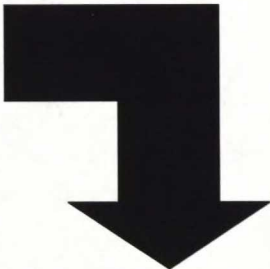
³ Only very few clusters are excluded based on this.

Example of forming the cluster

Example of forming the cluster is displayed below in figure 1. Total of 19 register entries, of which 11 are exchange trades, are compressed into four clusters. The first cluster contains initial entries into register on 1.1.1995.

Transaction Data

Reg. Date	ISIN Code	Amount	Buy/Sell	Price	Type	Ref. Date
1.1.1995	FI0009000483	750	10	0	0	
1.1.1995	FI0009000566	250	10	0	0	
1.1.1995	FI0009000582	1134	10	0	0	
1.1.1995	FI0009004865	300	10	0	0	
1.1.1995	FI0009800320	866	10	0	0	
1.1.1995	FI0009900336	2415	10	0	0	
19.5.1995	FI0009900336	200	10	35	1	15.5.1995
23.5.1995	FI0009900336	300	10	35	1	16.5.1995
13.9.1995	FI0009000582	60	20	135	1	7.9.1995
13.9.1995	FI0009000582	200	20	135	1	7.9.1995
14.9.1995	FI0009000582	70	20	136	1	8.9.1995
19.9.1995	FI0009000483	50	10	100	1	13.9.1995
19.9.1995	FI0009000483	200	10	100	1	13.9.1995
19.9.1995	FI0009004865	300	20	51	1	13.9.1995
19.9.1995	FI0009800320	200	10	110	1	13.9.1995
29.9.1995	FI0009000566	50	10	461	1	25.9.1995
3.4.1997	FI0009800643	200	10	63	52	
10.4.1997	FI0009003727	546	10	141	51	
22.4.1997	FI0009004824	200	10	54	1	17.4.1997



Clusters

Start Date	End Date	Duration	Cluster Type
1.1.1995	1.1.1995	0	Disqualified
15.5.1995	16.5.1995	1	Pure Buy
7.9.1995	25.9.1995	12	Rebalancing
3.4.1997	17.4.1997	10	Disqualified

Figure 1 – Example of Clustering. 19 Entries of a single investor in FCSD are compressed into four clusters according to the rules set up in this section. First column of the transaction data is the date on which the trade was entered into register. Type column tells whether transaction took place in the exchange (in which case the value is 1). The last column contains the date on which the trade was made in the exchange.

In the second cluster the investor has purchased 500 shares of Lemminkäinen in two transactions in consecutive days. Note that I have used the dates on which

the trades were made instead of the register entry dates as the start and the end date for the cluster.

In the third cluster the investor has at first sold 330 shares of Kymmene on Thursday and Friday, then on next Wednesday bought 250 shares of Instrumentarium and 200 of Orion A and sold 300 shares of Espoon Sähkö. After this there is an eleven-day pause, after which the cluster is completed on Monday by the purchase of 50 shares of Kone. Total value of the purchases is 70,050 FIM and that of sells 59,920 FIM, the ratio of buys to sells being 116.9%. Thus, this cluster is classified as rebalancing.

In the final cluster there are at first two non-exchange transactions. The investor has acquired 200 shares of YIT and 546 shares of Metra outside the exchange. The final transaction, purchase of 200 shares of Kemira, has taken place in the exchange. However, since there is not required 14 days break between this purchase and the non-exchange transactions, the entire cluster is excluded from the analysis.

Magnitude of the Rebalancing Activity

The first point of interest is whether there exists rebalancing transactions on the market, and if so, what is the magnitude of such activity. The overview is given in table 1.

	Cluster Type					Total
	Pure Buy	Buy inc. RB	Rebalancing	Sell inc. RB	Pure Sell	
No of Clusters	22554	2326	3073	3127	35563	66643
No of Trades	47414	11765	15698	15719	84182	174778
Total Value ^a	2,085,721	533,879	919,357	998,816	5,813,825	10,351,600

a. Values in '000 FIM

	Cluster Type					Total
	Pure Buy	Buy inc. RB	Rebalancing	Sell inc. RB	Pure Sell	
No of Clusters	33,8%	3,5%	4,6%	4,7%	53,4%	100,0%
No of Trades	27,1%	6,7%	9,0%	9,0%	48,2%	100,0%
Total Value	20,1%	5,2%	8,9%	9,6%	56,2%	100,0%

Table 1 – Summary of Clusters. Table 1 shows how 66,643 clusters formed from the 174,778 trades of the wealthy Finnish private investors in years 1995-2000 are divided between five categories. “Pure Buy” and “Pure Sell” categories contain only Buy/Sell transactions respectively. Rebalancing category contains clusters where the total value of buys is within the range of 75-125% of the total value of sells. Clusters in remaining two categories contain both buys and sells, but the buy/sell ratio does not fall into 75-125% range. All the values are counted using FCSD data. For “Rebalancing” and “Buy/Sell inc. RB” categories all the values are gross values, i.e. the values of buy and sell components are summed together.

Table 1 shows clearly that the investors do change the proportions invested in different stocks by trading, and the clusters containing these trades account for 23.7% of the value of all trading in the sample, 24.7% of all trades, and 12.8% of clusters. Numbers this large indicate that, in addition to liquidity needs, the will

to change the portfolio weights must also be taken into account in the analysis of the private investors behaviour.

Category "Rebalancing" is designed so that it contains those rebalancing clusters, which do not include major in- or outflows of funds. Due to frictions, lot sizes, and to allow for some in- and outflow there is a 25% margin. However, this category only accounts for some 38% of the value of the three categories including both buy and sell transactions (Rebalancing, Buy inc. RB, Sell inc. RB). Remaining 62% may indicate that many investors do rebalance only after liquidity needs (need to invest or divest funds) have led them to enter the stock market.

This result would be in contradiction to the Morton-Pliska model (1995), which suggests that investor should use in- and outflow of funds for rebalancing whenever possibly. It seems that investors frequently rebalance in precisely those situations, and are far less eager to rebalance when such opportunity does not exist. However, further analysis on this is needed.

Strange feature in this table is that it is strongly biased towards sells. This is not dependent on the sample, since the ratio of buys is approximately 43% percent in both the sample and among non-wealthy investors⁴. Thus, the effect of the investor lifecycle hypothesis can be excluded. I assume that the exclusion of purchases made in IPOs and SEOs bias the number of buys downward. The sells with these stocks take place in exchange, and thus these trades are included into sample.

Composition of the Clusters

It is interesting to see how many trades there are in a cluster, and what is the average value of a cluster. This question is addressed in table 2.

⁴ The exclusion of clusters containing intra-cluster trades changes this ratio downwards in table 1.

		Cluster Type					Total
		Pure Buy	Buy inc. RB	Rebalancing	Sell inc. RB	Pure Sell	
Panel A - Total Value of Trades^a	Valid N	22554	2326	3073	3127	35563	66643
	Mean	92	229	299	319	163	155
	Median	45	140	164	170	64	63
	Std Deviation	213	338	470	1,623	411	497
	Minimum	0	5	9	3	0	0
	Maximum	13,000	5,736	9,386	85,846	18,785	85,846
	Percentile 75	96	254	333	324	163	156
	Percentile 90	197	474	633	585	368	346
	Percentile 95	312	692	937	891	584	544
	Percentile 99	750	1,773	2,127	2,077	1,472	1,329
Panel B - No of Trades	Mean	2,10	5,06	5,11	5,03	2,37	2,62
	Median	1	4	4	4	2	2
	Std Deviation	1,89	3,11	3,47	3,27	2,45	2,59
	Minimum	1	2	2	2	1	1
	Maximum	52	31	42	33	50	52
	Percentile 75	2	6	6	6	3	3
	Percentile 90	4	9	9	9	5	5
	Percentile 95	5	11	12	11	6	7
	Percentile 99	10	16	18	18	13	13
Panel C - No of Stocks Traded	Mean	1,35	3,05	2,90	3,04	1,36	1,57
	Median	1	3	2	3	1	1
	Std Deviation	,86	1,42	1,41	1,46	1,08	1,20
	Minimum	1	2	2	2	1	1
	Maximum	16	14	15	14	25	25
	Percentile 75	1	4	3	4	1	2
	Percentile 90	2	5	5	5	2	3
	Percentile 95	3	6	6	6	3	4
	Percentile 99	5	8	8	8	6	7

a. All Values in '000 FIM

Table 2 -- Statistical Properties and Percentiles of Distribution of clusters -- 66,643 clusters formed from the 174,778 trades of the wealthy Finnish private investors in years 1995-2000 are divided between five categories. "Pure Buy" and "Pure Sell" categories contain only Buy/Sell transactions respectively. Rebalancing category contains clusters where the total value of buys is within the range of 75-125% of the total value of sells. Clusters in remaining two categories contain both buys and sells, but the buy/sell ratio does not fall into 75-125% range. Statistical properties and selected percentiles are presented separately for all cluster types. Panel A presents information on the total value of a single cluster, Panel B on the number of individual transactions in a cluster and Panel C how many different stocks have been traded in a cluster. All the values are counted using FCSD data.

Panel C of table 2 shows that in over 75% of buy and sell clusters investor trades with only one stock. However, this trading is often divided into two or more trades as demonstrated by larger values in panel B. Three categories including both buys and sells consist of more trades. It is interesting that in the clusters in these categories the mean of both *the number of trades* and *the number of stocks traded* is more than double compared to clusters in either buy or sell categories. This suggests that these clusters are not formed by chance from independent buy and sell clusters.

I tested whether the total value of the trades per cluster differs between categories with one-way analysis of variance (ANOVA). It indicated that the difference is significant at 0.1%-level ($F = 258.48$). When I excluded clusters with only one transaction (which do exist only in Buy and Sell categories), the F-value was somewhat smaller ($F=89.31$), but still highly significant at 0.1% level.

For the number of trades per cluster the results were even stronger. With single transaction clusters excluded $T\text{-value}=557.6$ (significant at 0.1%-level) and with them included $T = 2550$ (highly significant).

Between three categories in the middle there is a difference in the total value of trades per cluster with $F = 5.299$ (0.5%-level), but not for the number of trades ($F = 0.479$, highly insignificant).

The numbers in the upper end of the distribution are surprisingly large, especially since clusters containing intra-cluster trades are excluded. For sell cluster 1% contain at least 13 transactions, and 5% at least 6. For buy these numbers are 10 tr. (1%) and 5 tr. (5%), and for other three categories between 16-18 (1%) and 11-12 (5%).

Other way round, only 16.27% of the trades take place alone, and 48.44% in a cluster of three trades or less. This is presented in Table 3. The result means that the commonly used analysis based on the transaction data without taking clustering into account can possibly lead to strongly biased results.

		No of transactions		
		Frequency	Percent	Cumulative %
Number of transactions in cluster	1	28435	16,27 %	16,27 %
	2	31244	17,88 %	34,15 %
	3	24987	14,30 %	48,44 %
	4	20268	11,60 %	60,04 %
	5	14740	8,43 %	68,47 %
	6	11412	6,53 %	75,00 %
	7	8603	4,92 %	79,92 %
	8	6520	3,73 %	83,65 %
	9	5328	3,05 %	86,70 %
	10	4120	2,36 %	89,06 %

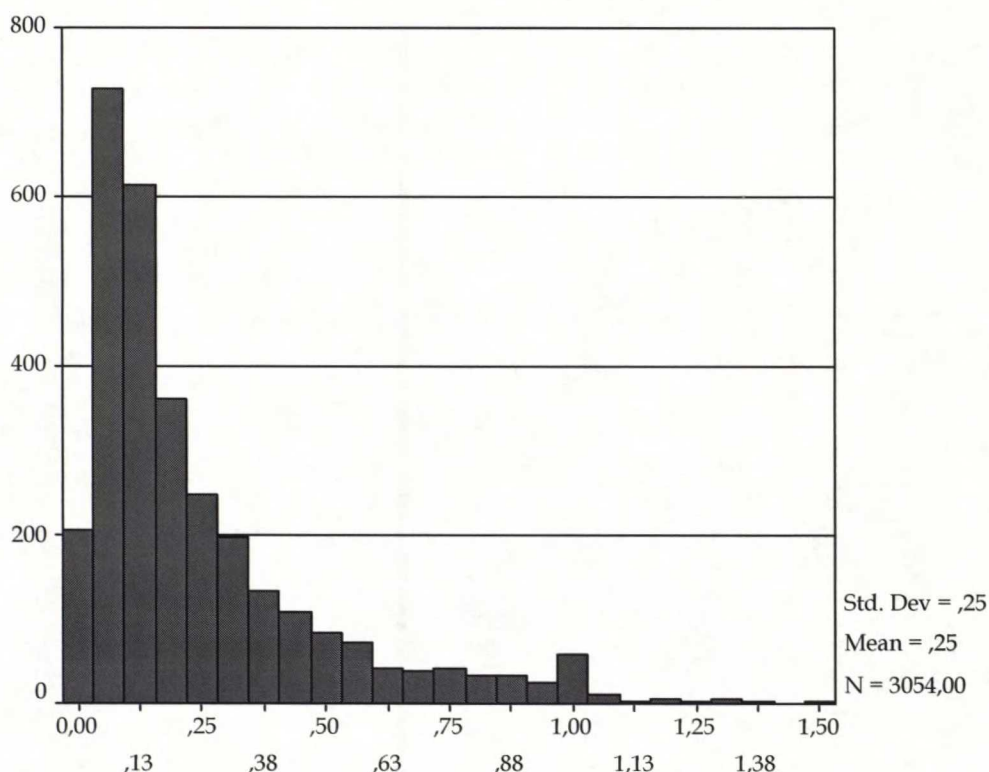
Table 3 -- The number of transactions by the number of other transactions in the same cluster -- 174,778 transactions of the wealthy Finnish private investors in years 1995-2000 are classified according to number of other transactions taking place in the same cluster. (Note: The bottom of the table is not displayed to make it more readable)

Practice of Rebalancing

In this section I try to describe how the portfolio rebalancing is done in practice. Important questions are 1) how large proportion of the portfolio is changed, 2) how rebalancing affects the number of stocks held 3) how long does the rebalancing take 4) how does the number of shares change during rebalancing and 5) how often do investors rebalance their portfolios.

How Large Proportion of the Portfolio is Rebalanced?

In the next figure the clusters are classified according to proportion (sold stocks)/(portfolio value before rebalancing). The figure should help us understand whether investors make major changes to their portfolios, or do they do only small adjustments, when they change weights by trading.



Proportion of Portfolio Sold

Figure 2 – Proportion of the portfolio changed during rebalancing – 3054 clusters of trades that are classified as “Rebalancing” are included in this figure. The proportion of the portfolio changed is on the x-axis, and the number of the cases on the y-axis. The proportion changed is counted as (total value of sells) / (total value of portfolio before rebalancing started). The usual adjustment is fairly small, median being 15.41%, mean 24.56% and standard deviation 24.90%. There is a peak in 100% indicating those cases in which the investor changes her portfolio entirely. Values over 100% percent are caused 1) by price appreciation between the moments on which the value of the portfolio was counted and the trading took place and 2) by the unlisted securities in the portfolio. I have excluded portfolios with this ratio exceeding 150% from the analysis. Value of sells is counted from FCSD data, the portfolio value using closing prices of the HEX on the day before the rebalancing started.

The figure shows that most common case is rather small adjustment, median being 15.41%. In 2/3 of all cases the proportion of the portfolio sold is under 24.20%. In some cases the entire portfolio has been changed, but these represent less than 3% of the clusters (proportion sold 97% or more).

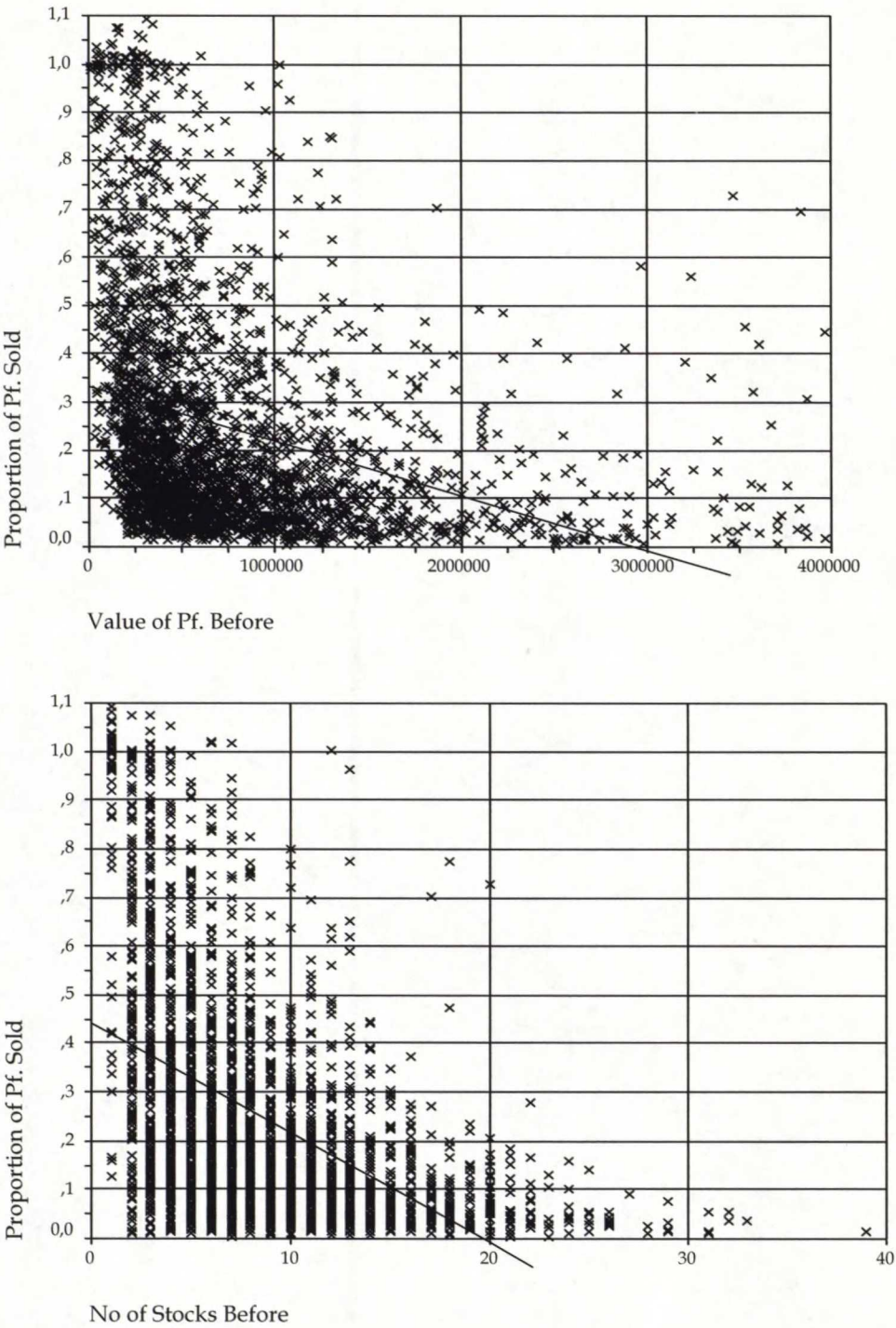
Different Factors Affecting the Proportion of the Portfolio Rebalanced

According to the theories presented earlier in this study, the value of the portfolio determines the number of the shares held (Brennan 1975). The number of the shares held determines the extent on which the portfolio can be/is diversified (CAPM model), and how quickly the actual portfolio weights diverge from the optimal ones (Morton and Pliska 1995).

In this section I study how these central parameters are interrelated, and related to the proportion of the portfolio that is changed during the rebalancing.

This sections starts with scatter diagrams sketching the relation between central variables and with the discussion on these relations. After that I present correlation between different variables in a table. The regression model of the variables concludes the section.

Figures 3a and 3b



Figures 3a and 3b – the relation between proportion of the portfolio sold during rebalancing and value of portfolio one day before rebalancing started (figure 3a) / the number of different shares in the portfolio one day before rebalancing started (figure 3b). 3054 clusters of trades by wealthy Finnish private investors in years 1995-2000 classified as “Rebalancing” are included in this figure.

Figure 3a demonstrates that the proportion of the portfolio sold during a rebalancing cluster is inversely related to the value of the portfolio before the first trade of the cluster. It also shows that the clusters, in which the value of the sells exceeds 50%, are concentrated on the smaller portfolios.

The figure 3b is modified so that the value of the portfolio before rebalancing started is replaced by the number of the stocks in the portfolio on that day. The number of stocks can be thought as approximation of portfolio diversification. It indicates that those investors who possess more diversified portfolios make smaller adjustments than those whose portfolios are less diversified. I.e. the proportion of the portfolio changed is strongly inversely related to the number of the shares in the portfolio. Potential reasons for this include at least

- 1) Lesser need to improve diversification of the portfolio
- 2) The portfolio weights change slower than in less diversified portfolios and even large proportional change in the value of one stock does not raise the need to rebalance
- 3) The investor handles his investments as a gambler chips on the roulette table: she either keeps her bet on one stock or changes it to another, whereas the financial theory suggests that an investor should often move only part of that bet. The proportion of a single bet to the total value of the portfolio is simply smaller on those portfolios, which consist of more stocks. The reason for the action in gambling hypothesis is however the same regardless of the number of stocks an investor holds

The rationale behind using number of stocks as a proxy for portfolio diversification is displayed in figure 4. It plots the relation between the proportion of portfolio's value held in a single stock and the number of the stocks in the portfolio. The proportions held on one single stock are large on any level, but the figure has clear downward slope. Using the number of stocks as a proxy naturally omits the effect of the covariances.

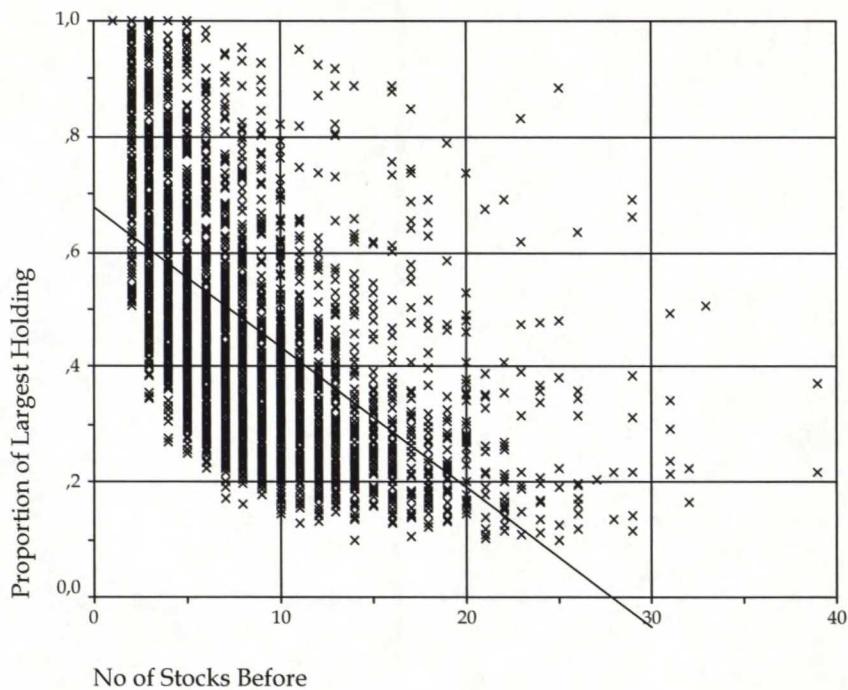
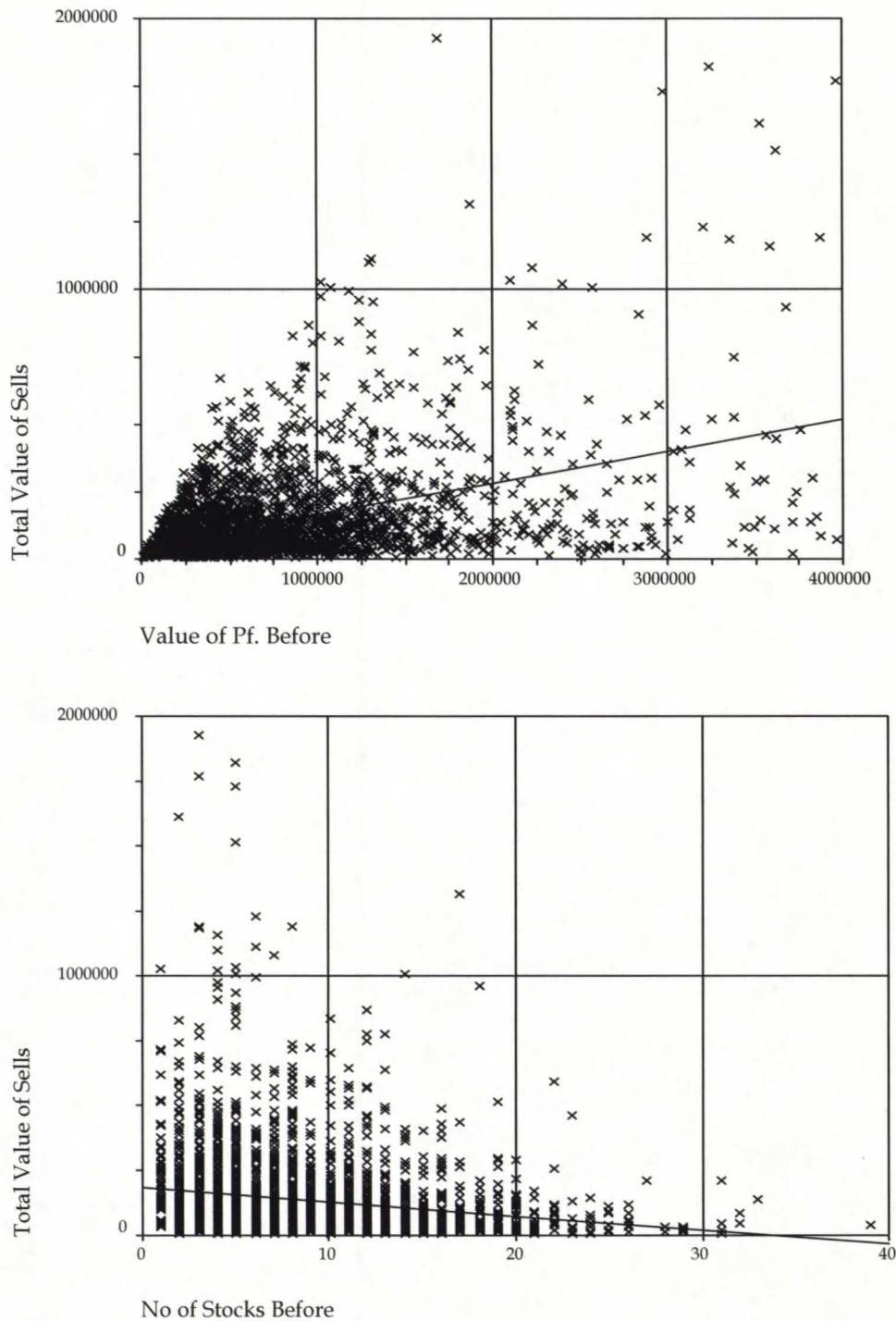


Figure 4 – Relation between number of stocks in the portfolio and the value of the largest single holding in the portfolio. The values are counted for 3054 cases, where the rebalancing of the portfolio has started on next day. The figure demonstrates that on average, the number of stocks is inversely related to the proportion of portfolios value held on any single stock.

In figures 5a and 5b the y-axes of the figures 3a and 3b are replaced by the total value of the sells during the rebalancings.

Figures 5a and 5b



Figures 5a and 5b – the relation between total value of sells during rebalancing and value of portfolio one day before rebalancing started (figure 5a) / the number of different shares in the portfolio one day before rebalancing started (figure 5b). 3054 clusters of trades by wealthy Finnish private investors in years 1995-2000 classified as “Rebalancing” are included in this figure.

Figure 5a demonstrates that the relation between total value of the portfolio and the value of the sales is positive, as might have been expected.

Figure 5b is somewhat more surprising. It shows that the FIM-amount the investors sells during the rebalancing is negatively correlated to the number of different stocks she holds. This is surprising, because the value of the portfolio (which is positively correlated to value of the sales) is positively correlated to the number of stocks held, as shown in the figure 6. This positive relation has earlier been documented by Karhunen and Keloharju (2001).

Figure 6

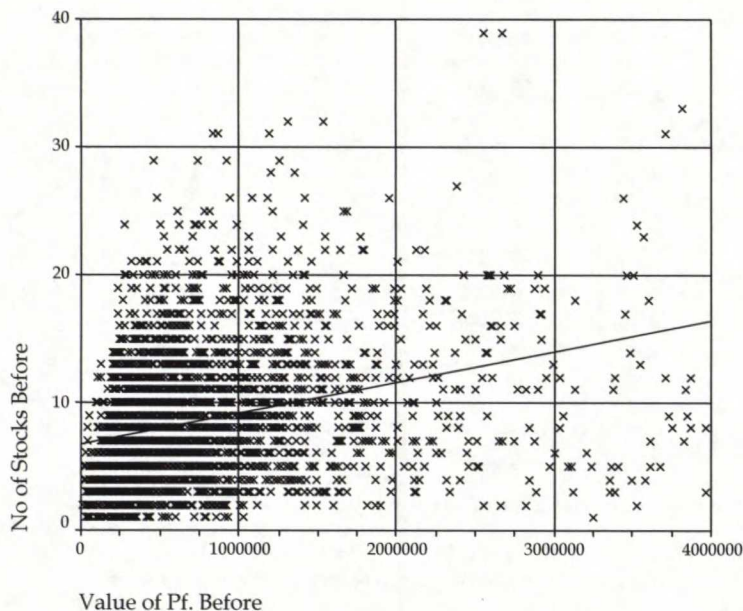


Figure 6 – Number of stocks compared to total value. Number of stocks the investor held in her portfolio is on y-axis, and the total value of the portfolio on the x-axis. The values are counted for 3054 cases, where the rebalancing of the portfolio started on the next day. The figure demonstrates that on average, the number of stocks is positively related to the total value of the portfolio.

Reason for this inverse correlation between no of stocks in the portfolio and the total value might be gambling-hypothesis formed earlier in this chapter. The number of stocks in the portfolio, though positively related to the total value of the stocks in the portfolio, is negatively related to the total value of holdings in any single stock (see figure 7). This means that if an investor would always sell

her entire holdings on a one stock, and trade only one stock at a time, the relation would look like that shown in figure 5b. Naturally, this is only a simplification to demonstrate the idea behind hypothesis.

Figure 7

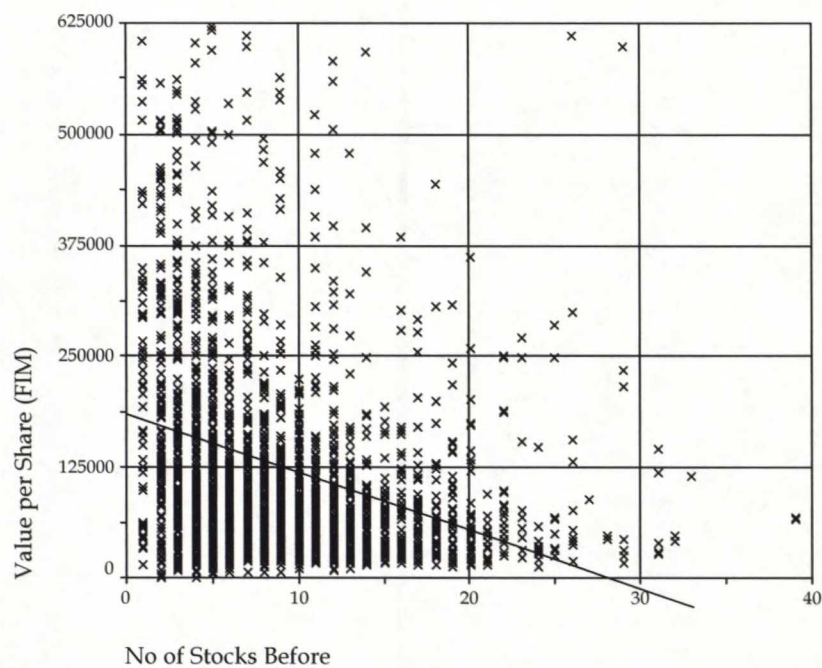


Figure 7 – Number of stocks compared to average value of holding in one share. Number of stocks the investor held in her portfolio is on y-axis, and the total value of the portfolio divided by the number of shares is on the x-axis. The values are counted for 3054 clusters of trading by the Finnish wealthy private investors in years 1995-2000, where the rebalancing of the portfolio started on the next day. The figure points out that on average, the number of stocks is negatively related to the total value held in a single stock. Note that the scale of the y-axis differs from that of the other tables presenting FIM-values.

It is important to note that figure 7 might be biased by the selection of the sample for this study, and thus may not be generalised. Especially this is the case with the upper limit (900,000 FIM) set for the value of the portfolio in 1st of January 1995. E.g. an investor who held 10 positions worth 200,000 FIM each is excluded from the analysis, whereas an investor with 4 such positions is included.

In table 4 I present the correlations between different variables in this subsection.

Correlations					
		No of Stocks Before	Total Value of Sells	Value of Pf. Before	Proportion of Pf. Sold
No of Stocks Before	Pearson Correlation	1,000	-,095**	,258**	-,460**
	Sig. (2-tailed)	,	,000	,000	,000
	N	3054	3054	3054	3054
Total Value of Sells	Pearson Correlation	-,095**	1,000	,290**	,367**
	Sig. (2-tailed)	,000	,	,000	,000
	N	3054	3054	3054	3054
Value of Pf. Before	Pearson Correlation	,258**	,290**	1,000	-,192**
	Sig. (2-tailed)	,000	,000	,	,000
	N	3054	3054	3054	3054
Proportion of Pf. Sold	Pearson Correlation	-,460**	,367**	-,192**	1,000
	Sig. (2-tailed)	,000	,000	,000	,
	N	3054	3054	3054	3054

** Correlation is significant at the 0.01 level (2-tailed).

Table 4 –Correlations between selected variables. This variables are used in this section to study the rebalancing, and are counted from same 3054 clusters of trades by the Finnish wealthy private investors in years 1995-2000 as all the figures. All the correlations are statistically significant at more than 0,1%-level. 19 cases from the original sample of 3073 are excluded, because “proportion of portfolio sold” exceeded 1.50 indicating incomplete information.

I have conducted the regression analysis for these variables using this same setting. The results are presented in table 5. Note that *the total value of sells* is omitted because it can be derived from other variables (i.e. *total value of sells = value of portfolio before * proportion of pf. sold*).

		Coefficients ^a						
		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
Model		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	,438	,008		56,909	,0000	,423	,453
	Value of Pf. Before	-1,214E-08	2,586E-09	-,078	-4,693	,0000	-1,721E-08	-7,066E-09
	No of Stocks Before	-,021	,001	-,440	-26,549	,0000	-,022	-,019

^a. Dependent Variable: Proportion of Pf. Sold

Table 5 – Linear regression $y = a + b_1x_1 + b_2x_2$, where y is Proportion of portfolio sold during rebalancing, x_1 is value of portfolio one day before rebalancing started and x_2 is no of different stocks in the portfolio before rebalancing. The regression is estimated from 3054 clusters of trades by the wealthy Finnish private investors in years 1995-2000 classified as “Rebalancing”. Adjusted R^2 of the model is 0.217 indicating relatively good fit. Correlations between variables are presented in table 4.

Table 5 indicates that both variables are statistically highly significant (0.1%-level) in this model, and both are negatively correlated to the proportion of the portfolio sold. However, the *number of stocks in the portfolio before rebalancing* has larger effect (standardized beta = -0.440) than the *value of the portfolio before rebalancing* (standardized beta = -0.078) on the *proportion of portfolio sold during rebalancing*.

Durations of the Clusters

An interesting question is how many days does it take to complete a cluster of trading. In other words, do the investors act rapidly completing the predetermined trades, or do they for example sell some stocks at first, and then think how to invest the proceeds? This distribution of durations is presented in table 6.

		Cluster Type										Total	
		Pure Buy		Buy inc. RB		Rebalancing		Sell inc. RB		Pure Sell		Count	Col %
		Count	Col %	Count	Col %	Count	Col %	Count	Col %	Count	Col %		
Duration in Tr. Days	0	17,685	78,4%	340	14,6%	747	24,3%	364	11,6%	26,789	75,3%	45,925	68,9%
	1	1,635	7,2%	261	11,2%	440	14,3%	256	8,2%	3,324	9,3%	5,916	8,9%
	2	664	2,9%	185	8,0%	268	8,7%	231	7,4%	1,282	3,6%	2,630	3,9%
	3	431	1,9%	188	8,1%	235	7,6%	258	8,3%	786	2,2%	1,898	2,8%
	4	354	1,6%	168	7,2%	210	6,8%	250	8,0%	635	1,8%	1,617	2,4%
	5	331	1,5%	169	7,3%	185	6,0%	242	7,7%	474	1,3%	1,401	2,1%
	6	269	1,2%	131	5,6%	156	5,1%	214	6,8%	436	1,2%	1,206	1,8%
	7	272	1,2%	139	6,0%	144	4,7%	216	6,9%	378	1,1%	1,149	1,7%
	8	201	,9%	124	5,3%	118	3,8%	197	6,3%	347	1,0%	987	1,5%
	9	198	,9%	135	5,8%	131	4,3%	192	6,1%	327	,9%	983	1,5%
	10	167	,7%	121	5,2%	108	3,5%	174	5,6%	253	,7%	823	1,2%
	11	80	,4%	63	2,7%	62	2,0%	82	2,6%	132	,4%	419	,6%
	12	58	,3%	68	2,9%	56	1,8%	82	2,6%	92	,3%	356	,5%
	13	46	,2%	39	1,7%	40	1,3%	75	2,4%	78	,2%	278	,4%
	14	34	,2%	54	2,3%	46	1,5%	76	2,4%	68	,2%	278	,4%
	15	37	,2%	41	1,8%	36	1,2%	60	1,9%	47	,1%	221	,3%
	16	31	,1%	30	1,3%	22	,7%	39	1,2%	33	,1%	155	,2%
	17	26	,1%	25	1,1%	14	,5%	39	1,2%	24	,1%	128	,2%
	18	21	,1%	14	,6%	17	,6%	36	1,2%	16	,0%	104	,2%
	19	8	,0%	14	,6%	19	,6%	24	,8%	18	,1%	83	,1%
	20	6	,0%	17	,7%	19	,6%	20	,6%	24	,1%	86	,1%
Total		22,554	100,0%	2,326	100,0%	3,073	100,0%	3,127	100,0%	35,563	100,0%	66,643	100,0%

Table 6 – Durations of the clusters – 66,643 clusters formed from the 174,778 trades of the wealthy Finnish private investors in years 1995-2000 are classified in five categories. Clusters are further classified according to the time between first and last day of the cluster measured in trading days. The proportion of clusters with certain duration to total number of clusters in that category is presented in the Col%-column. Note that the clusters with durations exceeding 20 days are excluded from this study.

Several conclusions can be drawn from this table. Firstly, the clusters are not formed from randomly distributed trades, which had coincidentally taken place close to each other. This can be seen from the three categories in the middle. If the trades were distributed randomly, these distributions would have been flat. Now the shorter time intervals clearly dominate.

Secondly, clusters including both sells and buys are completed slower than those with only one type of trades. This result holds also after including only those buy and sell clusters, which consist of more than one trade (clusters with one

trade are always completed in 0 days, three other cluster types always include more than one trade). Of the clusters consisting of at least two trades, 55.4% (6056) of buys and 53.2% (9983) of sells were completed in 0 days (table not presented here).

Thirdly, in the “Rebalancing” and “Rebalancing related” clusters, the time between first trade and last trade is rather long. This may suggest that many of these clusters are “sell induced”, i.e. that an investor has sold her stocks at first, and only after that decided what to do with the proceeds.

I have also checked the regression where total value of trades and number of trades explain duration. I have included only those clusters, which consist of at least 2 trades. The results indicate that the value of the trades does not have any explaining power when the number of trades is taken into account. On the contrary, the number of the trades is highly significant, and explains roughly 26% of the duration ($R^2 = 0.264$).

When the number of stocks traded (i.e. how many different stocks are either sold or bought in a cluster) is included into analysis, adjusted R^2 increases to 0.305.

These figures mean that the private investors do not need to, or they simply do not, take into account market impact of their trading. This impact was claimed to be reason behind the duration of the money managers buy or sell series by Chan and Lakonishok (1995). Some other factor(s) must explain the behaviour of private investors.

Number of the Shares and the Rebalancing

Next I turn on to examine the relation between the number of different stocks and the rebalancing process. Are the investors diversifying their portfolios, changing the weights between their current holdings, or are they ceasing holding some stocks altogether?

How does the number of stocks change during rebalancing?

In table 7 we can see how the number of the stocks changes during rebalancing.

No of New Stocks * No of Ceased Stocks Crosstabulation ^a

Count

		No of Ceased Stocks										Total
		0	1	2	3	4	5	6	7	8	12	
No of New Stocks	0	367	463	106	19	8	5	1	1			970
	1	495	837	155	46	10	3	3	1		3	1553
	2	96	166	89	17	6	1	2				377
	3	33	40	30	11	2						116
	4	10	8	13	3	1		1		1		37
	5	3	8		3	1						15
	6	2	1									3
	8	1							1			2
	Total	1007	1523	393	99	28	9	7	3	1	3	3073

a. Above the diagonal are the cases, in which the number of stocks has decreased (n=854, 27.29%), on the diagonal (shaded) number has not changed (n=1305, 42.47%), and beneath the diagonal the number of stocks held has increased n=(914, 29.74%).

Table 7 – Number of new stocks purchased and old stocks divested during rebalancing – 3073 clusters of trades that are classified as “Rebalancing” are included in this table. Cells in upper right part present increase in the total number of stocks, in lower left the number has decreased. When interpreting the table, note that this table does not present trades on those stocks that are neither ceased nor new ones, i.e. one cluster in cell (1 , 1) may contain trades with only 2 stock, another cluster in the same cell trades with 5 different stocks.

We find that by far most common case is to cease holding 1 stock and to purchase 1 new stock, with 27.23% of cases. Natural interpretation is that these cases indicate speculation, in which an investor moves her assets from one stock, for which she expects returns to be unsatisfactory, to one with better prospects.

On the top row excluding the leftmost cell are 603 cases (19.62%). In these cases an investor is divesting her portfolio and investing the proceeds to the stocks she already held. Reason for this behaviour can only be speculated here. I find it likely that an investor has strong trust on a past winner in her portfolio, or she thinks that one of the past losers starts to outperform the market. This is also supported by the fact, that in 516 of these cases (85.57%) the proceeds where

invested in only one stock, and in 65 cases (10.78%) in two stocks (table not presented here).

These cases are interesting since the investor is almost certainly increasing the variance of the portfolio, presumably in the hope for the better mean. This seems to be against the theories presented in the theoretical part, especially that of Brennan's (1975). He claimed that investors with small value portfolios hold poorly diversified portfolios because of the transaction costs associated with the diversification. Table 7 suggests that the investors are sometimes actually willing to pay transaction costs to reduce the diversification.

On the leftmost column (excluding topmost cell) is the opposite case ($n=640$, 20.82%). Here the investor is almost certainly increasing the diversification of the portfolio and decreasing the variance of the return.

Table 7 supports the hypothesis that investors treat their holdings in one stock as an entity. They only rarely sell parts of this entity, but very often whole of it. This is demonstrated by the fact that only in 32.77% percent of the clusters an investor has not ceased holding any of the stocks.

Number of the stocks in the portfolio and the change in that number

Next I will take a look at how the number of stocks originally held affects the change in the number of the stocks. If an investor needs to rebalance her portfolio for some reason, and pay the related transaction costs, it might be expected that the investors are more eager to diversify their portfolios when the number of stocks is small. As the number of stocks gets larger, the gains from further diversification diminishes, and consequently she switches weights, or ceases holding some of old stocks when acquiring new ones.

No of Stocks Before * Change in No of Stocks Crosstabulation

Count		Change in No of Stocks													Total
No of Stocks Before		-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	
							1	32	27	4	2				66
1	2						22	66	48	5	3		1		145
2	3					7	46	107	52	14	4	1			231
3	4				1	9	56	104	58	14	4	3			249
4	5			1	3	12	58	120	69	17	4	2	1	1	288
5	6		1		2	13	52	100	60	10	7		1	1	248
6	7			1	2	9	59	130	65	6	2	4			278
7	8			2	4	18	54	95	50	14	1	2			240
8	9		1		2	8	46	83	60	9	1		1		211
9	10		1	2	2	19	49	73	35	21		3			205
10	11			2	3	15	35	74	32	10	1	1			173
11	12	1	2	1	2	12	26	65	28	4	3				145
12	13		2	3	1	6	22	59	21	8	1	1			125
13	14					6	22	37	20	4	4				93
14	15			1	2	5	19	28	8	2					65
15	16	1				4	12	26	14	3	1				61
16	17			1	1	1	9	20	12	1					45
17	18				1	4	6	20	7						40
18	19				2	1	11	14	7						35
19	20					2	7	15	8	1	1				34
20	21		1			3	2	8	3						17
21	22				1	1	4	7	4			1			18
22	23				1		2	3	3	1					10
23	24					1	2	4	2	1	1				11
24	25						4	2	1	1					8
25	26					1	1	5	2	1					10
26	27							1							1
27	28							2							2
28	29					1	2	3		1					7
29	31					1	3	1							5
30	32						2								2
31	33						1								1
32	39						1	1							2
33	41										1				1
34	43						1								1
Total		2	8	14	30	159	637	1305	696	152	41	18	4	2	3073

Table 8a – Change in the number of stocks during rebalancing – 3073 clusters of trades that are classified as “Rebalancing” are included in this table. On rows is the number of stocks before rebalancing started, in columns the change in that number. Note that right and leftmost columns are not shown to make the table more readable.

Table 8a seems to be fairly symmetric, and does not reveal any clear patterns to support or reject the hypothesis. The same table in compressed form is presented below (Table 8b).

Change in the number of stocks					
		Decrease	No Change	Increase	Total
No of Stocks Before	1-3	76	205	161	442
	4-6	208	324	252	784
	7-9	206	308	215	729
	10-12	172	212	138	522
	13-15	89	124	69	282
	16-18	40	66	38	144
	19-21	29	37	20	86
	22-24	12	14	13	39
	25-	18	15	7	40
	Total	854	1305	914	3073

Larger of decrease / increase bolded

Table 8b – Compressed version of table 8a

Table 8b indicates that clusters with an increases in the number of stocks seem to dominate in portfolios consisting of less than 7 stocks, whereas opposite seems to be true for portfolios consisting of more than 10 stocks.

The chi-square-test for table 8b (no change column excluded) tells that change in the number of stocks is dependent on the number of stocks before rebalancing. This is however driven by the first two rows. With 1st row included, the relation is significant on 0.1 %-level, and with 1st-excluded (but 2nd included), the significance decreases onto 5%-level. With 1st and 2nd row excluded the relation is statistically insignificant.

Based on the chi-square test and table 8a, it can be said that when rebalancing, the investors are more willing to increase the number of stocks in their portfolio, when it consists of less than 7 stocks. After this level, the investors seem to be as likely to decrease as to increase the number of stocks.

How Often an Investor Rebalances Her Portfolio?

The interval between rebalancings is a central question in theoretical models. In the models presented in the theoretical part it varies from infinitesimal small to up to seven years. The number of rebalancings per investor and the number of other clusters completed by these same investors is presented in table 9.

No of rebalancing clusters												
	0	1	2	3	4	5	6	7	8	9	10-	Total
No of Investors	N=11207	N=1176	N=385	N=161	N=66	N=32	N=11	N=9	N=5	N=2	N=3	N=13057
% of Investors	85,83%	9,01%	2,95%	1,23%	,51%	,25%	,08%	,07%	,04%	,02%	,02%	100,00%
Rebalancing	,00	1,00	2,00	3,00	4,00	5,00	6,00	7,00	8,00	9,00	11,00	,24
Buy/Sell inc. RB	,25	1,05	1,72	2,43	2,11	2,56	4,45	4,11	5,80	2,00	3,00	,42
Buy/Sell	3,89	7,09	9,15	9,17	8,92	9,03	9,00	7,67	8,80	4,50	10,00	4,45
Tot. No of Clusters	4,15	9,14	12,87	14,60	15,03	16,59	19,45	18,78	22,60	15,50	24,00	5,10

Table 9 – Number of rebalancing clusters and the relation between the number of these clusters and other clusters per investor. In the table 13057 wealthy Finnish private investors are divided according to the number of rebalancing clusters they completed during years 1995-2000 (in columns). The average number of other cluster types per investor is counted, and these are presented in rows. In the bottom row is the total average number of clusters per investor. Table indicates that there is a strong positive correlation between the number of rebalancing clusters and other types of clusters.

From table 9 we see that only some 14.17% of the investors rebalanced their portfolio during the five-year period. Compared to the theory, this number seems to be small. It is possible that those investors, who did not rebalance, used other trades to keep their portfolio weights near suitable levels, as Morton and Pliska (1995) suggested.

The investors, who have not rebalanced at all, completed on average a total of 4.15 clusters of trades. This volume of trading should be enough to keep the portfolio weights near optimal levels, if the investors act according to Morton-Pliska model.

Table 9 demonstrates a strong and significant positive correlation between number of rebalancing clusters and other types of clusters. This means that those investors who are most active to rebalance their portfolio by trading also trade the most. This result is in sharp contradiction to Morton-Pliska model, because they have had plenty of opportunities to move the portfolio weights onto right

levels without costly rebalancing trades. Thus, the reason behind many of the rebalancing clusters is likely to be unrelated to mean-variance optimisation.

Interval between rebalancings

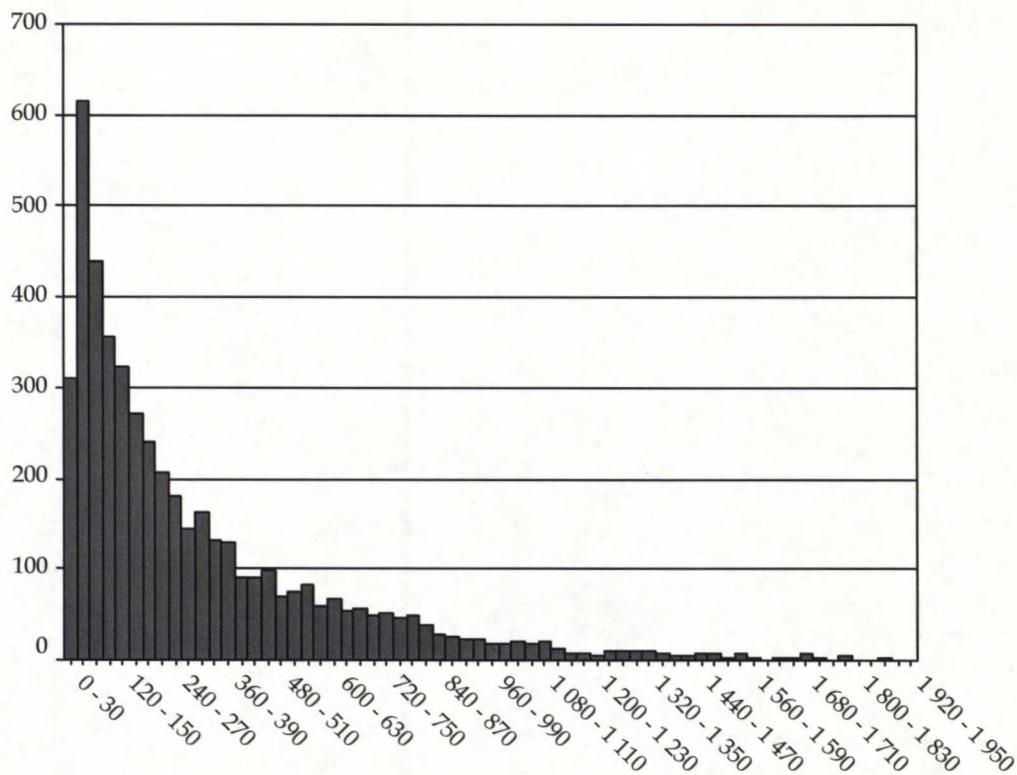
This section concentrates on the interval between rebalancings. Theory indicates that the rebalancing is caused by portfolio weights drifting too far from the investor's target when asset prices change. Other reason to cause increase in the gap between target and the portfolio actually held is that investor changes her target.

Investors' target mixes and preferences are unknown. In addition, every individual investor has so few clusters that it is impossible to determine their underlying distribution.

For these reasons I concentrate on two questions:

- 1) How long is the average interval between rebalancings, and what is the distribution? This is useful information for financial service institutions offering services.
- 2) Are there frequent intervals? Common belief is that the household investors manage their portfolios with frequent intervals, for example once a year during the holidays. If this is the case, this could be observed from the data.

These questions are addressed in figure 8.



Interval Between Clusters in Natural Days

Figure 8 – Intervals between investors rebalancing or rebalancing related clusters– For each wealthy Finnish private investor in the sample, who had at least 2 trading clusters classified as “Rebalancing” or “Rebalancing including Buy/Sell” during years 1995-2000, the interval between those clusters in natural days is counted. (N=4836, mean = 307.36, st.dev = 317.20, median = 190, mode = 28)

There are some important points to note when interpreting this figure. Firstly, the interval between clusters cannot be less than 14 days (+duration of the first cluster), due to the definition of the cluster. Secondly, since the length of the dataset is 5 and half years, and to be included in this figure there needs to be at least 2 rebalancing related clusters, there is a selection bias towards shorter intervals. I however assume that it affects the results only for several year (>2) intervals and thus does not change the results qualitatively.

The distribution of intervals in figure 8 is strongly skewed to the left. Short intervals dominate. This indicates that the clusters are clustered together. When

an investor has entered the market and completed a rebalancing/rebalancing related cluster, she is much more likely to complete another one after relatively short interval.

This may indicate that there are costs incurred before first transaction. Dumas and Luciano (1991) mention among transaction costs cost of analysis, information search cost and any expense incurred in the process of deciding upon and placing an order. It is natural to augment this list with the cost that is associated with the effort of finding out how the transaction is done in practice (e.g. where, how and for whom to place an order etc.).

The clustering of clusters is in line with the results of Barber and Odean (2002). They study how the household investors behave after switching from phone based to online-trading. They show that after the first trade completed online the investors accelerate their trading. After the switch, the turnover of the portfolio jumps for few months before settling on to higher-than-pre-switch level. The ease of access thus increases trading.

The figure 8 does not provide any evidence on the hypothesis that the investors would manage their portfolios on frequent intervals.

Does Rebalancing Improve the Portfolio?

In this section I will at first study how the mean of the portfolio and the variance of the mean change during portfolio rebalancing. I calculate these measures for both pre- and post-trading portfolio of the investor. The properties of these portfolios are then compared with each other.

After this I combine these measures in Sharpe Ratio, and examine how does it change during the rebalancing.

I start my analysis by defining the portfolio weights for two portfolios; for one that the investor held just before rebalancing, and for another one that she held immediately after rebalancing ended. For the analysis I calculate the returns for

these portfolios using dividend and split adjusted return-data following the rebalancing. The reason for using post-trading data is that it is likely to be less biased than pre-trading returns.

Odean (1999) studies the pre- and post transaction returns for the stocks that the clients of a discount brokerage house traded. He documents large, positive, abnormal returns preceding the trade. The returns following the trades are more normal, though the returns of the purchased stocks under perform that of the sold ones statistically significantly.

Grinblatt and Keloharju (2001a) have also reported that the returns prior to the trade affect on the investors trading behaviour. They conclude that “ generally, high past returns make it more likely that a domestic [Finnish] investor will sell rather than buy a stock. This effect lasts for returns up to a week in the past for some of the investor categories and up to three months for households and foreigners.”

In addition to using post-rebalancing returns, I also try to ensure the non-biasness of the results by skipping the returns of the five trading days immediately after the cluster. It is easy to imagine situations in which some event potentially having price impact may increase trading on the stock prior to that event (and prior to the price impact). Release of the interim report is an example of such situation.

The portfolio returns are determined for 30 three-trading-day periods starting from the day 6 after rebalancing ended (i.e. 1st period includes trading days 6, 7 and 8, 2nd days 9-11, and 30th days 93-95). Same portfolio weights are used for every period. The three-day period is chosen as a compromise between differing aims:

- 1) *To keep the period used short. This is important to minimise the number of stocks that cease to be quoted during the period, and to minimise the effect of the changes in the correlations and returns of the stocks.*
- 2) *To have enough observations to have statistically meaningful estimate of the return and the variance.*

- 3) To keep the period for individual observation long enough to minimise the effect of the infrequently traded stocks.

The details of counting returns and their variances are presented in the appendix.

The Return and the Variance

In the table below I have summarised returns and the standard deviations of the returns for different cluster types.

		Cluster Type ^a					Total
		Pure Buy	Buy inc. RB	Rebalancing	Sell inc. RB	Pure Sell	
Return Before	Valid N	N=3070	N=1566	N=2037	N=2088	N=4603	N=13364
	Mean	,00239	,00180	,00212	,00232	,00304	,00249
	Std Deviation	,00682	,00714	,00703	,00713	,00772	,00727
Return After	Mean	,00244	,00203	,00230	,00230	,00303	,00255
	Std Deviation	,00682	,00721	,00713	,00702	,00768	,00726
Change in Return	Mean	,00005	,00023***	,00018**	-,00003	-,00001	,00006
	Std Deviation	,00170	,00284	,00307	,00278	,00214	,00242
St. Dev Before	Mean	,03143	,03087	,03089	,03191	,03330	,03200
	Std Deviation	,01451	,01512	,01482	,01613	,01584	,01538
St.Dev After	Mean	,03114	,03134	,03145	,03173	,03350	,03211
	Std Deviation	,01428	,01529	,01504	,01646	,01583	,01544
Change in St.Dev	Mean	-,00029***	,00047**	,00055***	-,00019	,00020*	,00011
	Std Deviation	,00388	,00602	,00742	,00682	,00525	,00574

a. 1-sample t-test, difference from 0 significant on * = 5%-level, ** = 1%-level, *** = 0.1%-level. Test conducted only for change in return and change in standard deviation for individual groups. The total is omitted because Pure Sell and Pure Buy contain only samples of the entire groups.

Table 10 – Returns and Standard Deviations of Returns Counted for pre-and post-trading portfolios. The returns in this table are counted for pre- and post-trading portfolios (Return Before and Return After respectively) using 30 returns for 3-day periods from t+6 to t+95, t being the end date of the cluster. Standard deviation has been calculated for each of the portfolios, and these intra-portfolio standard deviations for the return are presented in the lower part of the table (names with “St.Dev”). The standard deviations associated with each of the six line items (i.e. Return Before, Return After etc.) are calculated between the portfolios belonging into that group and they are presented together with that line item (names with “Std Deviation”). Due to the computational effort required, the values are counted only for the sample of the “Pure Buy” and “Pure Sell”

clusters. Returns are not scaled in the table, but the mean return of 0.249% in three days would translate into 23.23% p.a.

There are some notes to be done before interpreting the table 10. Firstly, years 1995-2000 were exceptionally good in the Helsinki Exchanges, as demonstrated by mean raw returns ranging from 0.18 to 0.30% for three days for different categories (approx. 16%-29% p.a.). Secondly, the returns have been very volatile. Mean standard deviation of the return of the portfolio being 3.2% per three days, translating to 1.85% per day.

I ran a brief simulation to check how this volatility might affect the estimate of the return. With parameters mean = 0.212% and standard deviation = 3.2% I simulated 200 series consisting of 30 returns assuming that the returns are normally distributed⁵. The mean was counted for each of the series. The mean of these mean returns was 0.199%, 95% confidence interval being 0.018% to 0.380%. I thus think, that when all the groups consist of more than 1500 series of 30 observations, the approximation of the mean is reasonable good to conduct an analysis.

Compared to the Odean's results, table 10 shows surprisingly good performance from Finnish household investors when they trade. The mean of return does not decrease statistically significantly on any of the categories, whereas there is a increase in mean for "Rebalancing" and "Buy inc. Rebalancing" categories.

It is interesting that the mean of the returns differs considerably across different cluster types. The equality of the means can be rejected at the 0.1%-level (One-way ANOVA, $F=5.030$). Especially surprising is the return of the "Pure Sell"-category, which is 0.304% for pre-trading portfolio (0.303% for post-trading). These both greatly exceed the mean of all returns, 0.252%.

Grinblatt and Keloharju (2000) show that Finnish household investors followed contrarian strategies during years 1995-1996, selling past winners and buying

⁵ The return series usually are not normally distributed. Instead, they are more peaked and the tails are fatter. This means that the estimate of the error calculated here is likely to overstate the real error.

past losers. This contrarian strategy subsequently fared worse results than strategies followed by the more sophisticated investor categories.

This conclusion is supported by my results; the household investors selling stocks really held stocks that subsequently outperformed those held by other investors. However, even after sell, their portfolios continued outperforming portfolios of other household investors. It is possibly that this result is specific to this period, and related to the incredible boom in high technology sector. Further analysis is however outside the scope of this study.

The Sharpe Ratio

As seen in table 10, when investor completes a cluster classified as rebalancing, both the return of the portfolio, and the standard deviation of the return increase. Thus we cannot know whether the rebalancing improves the portfolio in the mean –variance sense without further analysis. I use Sharpe Ratio to conduct such an analysis.

The Sharpe Ratio is defined as the ration of the excess expected return to the standard deviation of the return:

$$SR = \frac{\mu - R_f}{\sigma},$$

Mean and the standard deviations are those calculated in the previous section. As a risk free rate I choose 1-year EURIBOR / HELIBOR. This rate was relatively stabile during the years 1995-2000, especially compared to the mean return of the portfolios. Therefore, and to minimise computational effort, I use the median of this return during the period, 3.9% p.a., for entire analysis.

The summary of the Sharpe Ratios is presented in table 11a.

	Valid N	N=2037
Sharpe Ratio Before	Mean	,08220
	Std Deviation	,21711
Sharpe Ratio After	Mean	,08572
	Std Deviation	,21659
Change in Sharpe Ratio	Mean	,00352
	Std Deviation	,08205

Table 11a – Sharpe Ratio for pre-and post-trading portfolios, and the change in that ratio. The returns and the standard deviation of the return for pre- and post-trading portfolios (Return Before and Return After respectively) have been calculated using thirty 3-day periods of returns from t+6 to t+95, t being the end date of the cluster. The Sharpe Ratio has been calculated for these portfolios. Total of 2037 clusters of trading by wealthy Finnish private investors in years 1995-2000 classified as rebalancing have been included in this table.

According to table 11a the mean-variance ratio measured by the Sharpe Ratio seem to have improved during rebalancing. However, this improvement is statistically insignificant (one sample t-test, t=1.938). Because the standard deviation is large, it is possible that large improvements / impairments in some portfolios hide the smaller effects in the majority of the portfolios. I have thus presented change in Sharpe Ratio in the non-parametric form in table 11b. The change has been replaced by -1 if it was negative, by 1 if positive.

Table 11b

+/- Change in Sharpe Ratio					
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid ^a	-1	1013	49,7	49,7	49,7
	1	1024	50,3	50,3	100,0
Total		2037	100,0	100,0	

^a. The changes in the Sharpe Ratio have been recoded, -1 presenting smaller and 1 higher sharpe ratio for the post-trading portfolio when compared to the corresponding pre-trading portfolio.

Table 11b – +/- Change in the Sharpe Ratio between pre-and post-trading portfolios. The numeric change in Sharpe Ratio has been replaces by -1 when negative, by 1 when positive. See table 11a for further explanation of this table.

Table 11b convincingly shows that the effect of trading on the Shape Ratio does not have any clear direction.

Despite of the results in table 11a and 11b, it is possible that there is a subgroup of household investors, which actually act according to the portfolio theory, and try to optimise the mean-variance ratio of their portfolio. Those investors, who switch weights in their portfolio for speculative purposes, may mask this group. The most likely candidate to be such group are in my opinion those invetors, who do not divest any of their old holdings totally, and invest the proceeds in stocks they already held.

For this reason, I have grouped the clusters according to the number of the new stocks purchased during rebalancing and the number of the stocks that were entirely divested during the rebalancing. The distribution of the rebalancing clusters divided this way has already been presented in Table 7. For each of these groups I have counted a proportion of the portfolios the Sharpe Ratio of which has improved to the total number of portfolios in that group.

Improvement Ratio =

No of Improved Portfolios

Total no of Portfolios

Results are presented in table 12.

		No of Ceased Stocks					
		0	1	2	3	Total	
No of new Stocks	0	Improvement ratio	0,448	0,497	0,421		0,469
		N	270	326	76		672
	1	Improvement ratio	0,503	0,486	0,489	0,536	0,493
		N	322	537	94	28	981
	2	Improvement ratio	0,543	0,659	0,544		0,590
		N	70	91	68		229
	3	Improvement ratio	0,550	0,600			0,578
		N	20	25			45
Total Improvement ratio		0,487	0,509	0,483	0,536	0,498	
Total N		682	979	238	28	1927	

Table 12 – Change in Sharpe Ratio between pre-and post-trading portfolios for different subgroups. The return and the standard deviation of the return for pre-and post-trading portfolios (Return Before and Return After respectively) have been calculated using thirty 3-day periods of returns from $t+6$ to $t+95$, t being the end date of the cluster. The Sharpe Ratio has been calculated for these portfolios. The ratio of portfolios the Sharpe Ratio of which has improved to total number of portfolios is presented in the table. Total of 2037 clusters of trading by wealthy Finnish private investors in years 1995-2000 classified as rebalancing are used for this table, but the ratio has not been counted for groups consisting of less than 20 portfolios.

Table 12 indicated that the improvement / impairment of the Sharpe Ratio of the portfolio might be related to the change in the number of stocks. In those groups where the number of new stocks is larger than the number of ceased stocks, the ratio is over 0.5 and vice versa (with one exception).

Within those 270 cases where the investor has only switched weights between her current holdings without divesting any stock entirely or acquiring any new stocks, there has been an improvement in Sharpe Ratio in 121 cases, and impairment in 149 cases. It seems unlikely that the investors, which would try to optimise the mean-variance ratio of their portfolio, would fare this poor. The aim behind these clusters is therefore likely to be something else.

IV. Summary and Conclusions

In this study I have studied the portfolio rebalancing of the wealthy private investors. The methodology is based on the idea of forming clusters from the trades conducted by individual investors temporally close to each other. This together with the information on the composition of the investor's portfolio on any day offers excellent opportunity to study the behaviour of these investors.

At the beginning of the study I have presented the commonly used modern portfolio theory, which is based on the idea of investors trying to maximise their expected utility by optimising the mean-variance ratio of their portfolio. This is used as a framework for the empirical part of the study.

The empirical part of the study shows that the trades of the household investors are strongly clustered. They conduct trades that look like rebalancing, i.e. sell holdings in one stock and use the proceeds to buy some other stock. Around 24.7% of the transactions take place in such cluster including both buys and sells.

However, I do not find any evidence that household investors would act according to the portfolio theory. Quite on the contrary, their behaviour seems to be highly speculative. They very often seem to treat their holdings in one stock as an entity. When they sell some of that entity, they very often sell all of it.

Thus, on average, the rebalancing activity does not have effect on the mean-variance ratio of the portfolio measured by the Sharpe Ratio. This does not exclude the possibility of some household investors optimising mean-variance ratio, though I fail to identify such group.

I have demonstrated that in roughly 20% of the clusters, where an investor switches her holdings between different stocks, she divests her holding in some stock(s) altogether and invests the proceeds to the stock(s) she already held. This effectively decreases the diversification of the portfolio, and seems to be against

the theories presented, especially that of Brennan's (1975). He claimed that investors with small value portfolios hold poorly diversified portfolios because of the transaction costs associated with the diversification. My findings suggest that the investors are sometimes actually willing to pay transaction costs to reduce the diversification.

Potential reason for this behaviour is the hypothesis of overconfident investors (see Odean 1999, Barber & Odean 2001, 2002). The hypothesis states that the overconfident investors "overestimate the precision of their knowledge about the value of a financial security. They may also overestimate the probability that their personal assessments of the security's value are more accurate than the assessments of others." (Barber and Odean 2002). The behaviour of the household investors observed in this study fits in this hypothesis.

In addition to the lack of the mean-variance optimising, another central finding in this study is that the trades of an investor are very strongly clustered together. Only some 16.3% of the trades take place alone, without any other trades during two weeks preceding or following the trade, whereas three or more other trades accompany roughly 40% of the transactions. This finding should be taken into account in the setting of the studies, which utilise transaction data where the investor conducting the trade is not specified.

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Appendices

Appendix A – Calculation of the Returns for the Portfolios

- 1) The portfolio weights for the portfolio are defined for the pre-rebalancing portfolio using prices on the day start of the rebalancing -1 and for post rebalancing portfolio using prices on the day end $+1$.
- 2) The returns are counted for 30 three-day-periods starting from day end+6, i.e. for total of 90 trading-days.
- 3) The return for stock i and period $[t, t+2]$ is counted as $r_{it} * r_{it+1} * r_{it+2}$
- 4) Returns for each period are multiplied by portfolio weights and summed;

$$\sum_i w_i r_i$$
, where w is portfolio weight.
- 5) If the return information for some stock is unavailable, that stock is omitted. Instead, the return of other stocks is scaled by dividing the sum of the return of the included stocks by the sum of their weights $\sum_i w_i$
- 6) 1 is subtracted from this return to obtain return in percentages (e.g. $1.0037-1=0.0037$)
- 7) After counting return for all 30 periods, the average is counted by summing all returns and dividing the sum by 30.

- 1) The variance is counted as $\frac{\sum_i \left(r_{i,t,t+2} - \bar{r} \right)^2}{30}$, where $r_{i,t,t+2}$ is return for one period and \bar{r} average return.

- 2) To qualify into analysis, the price and return information must be available for reasonable large proportion of the portfolio, and the pre- and post-rebalancing portfolios can not differ from each other too much in this sense. Thus I formulate three conditions that the portfolio must meet:

- Price date must be available for at least $\frac{3}{4}$ of the securities the investor holds.

- *The weight for which the return is counted (see point 5 above) must be at least 85% for each three-day period.*
- *The minimum weight for which the return is counted must not differ more than 5%-points between post and rebalancing portfolios. (E.g. although the portfolio would meet other conditions, it is excluded if the minimum weight is 91% for post-rebalancing portfolio and 98% for pre-rebalancing portfolio.)*